TRIVANDRUM

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**E – 1632**

Reg. No. : .....

Name : .....

**Sixth Semester B.Sc. Degree Examination, April 2018**

**First Degree Programme Under CBCSS**

**Mathematics**

**Core Course – XII**

**MM 1644 : GRAPH THEORY**

**(2013 Admn.)**

Time : 3 Hours

Max. Marks : 80

SECTION – I

**All** the first **10** questions are **compulsory**. They carry **1** mark **each**.

1. For what values of n is the complete graph $K_n$ Euler ?
2. Give example of an isograph.
3. What is the diameter of the circuit $C_n$ ?
4. What is the relationship between the number of edges of a simple G and the number of edges of its complement G′ ?
5. Draw the graph with incidence matrix $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$.
6. Suppose that a connected planar graph has 30 edges. If a plane representation of this graph divides the plane into 20 regions, how many vertices does this graph have ?
7. Give two non-isomorphic graphs with vertex degrees 2, 2, 2, 1, 1.
8. Prove or disprove : *There exists a complete bipartite graph with 5 vertices which is also a tree.*
9. What is meant by a directed Euler line in a digraph ?
10. Let G be the graph obtained by removing one edge from $K_{3,3}$. Draw a plane representation of G.

**P.T.O.**



## SECTION – II

Answer **any 8** questions from among the questions **11** to **22**. These questions carry **2** marks **each**.

11. Does there exist a graph with four edges and four vertices which have degrees 1, 2, 3, 4 ? If yes, draw such a graph, otherwise state why ?

12. Prove that no simple graph with two or three vertices is self-complementary.

13. If a graph G is disconnected, show that its complement is connected.

14. Explain Konigsberg bridge problem and draw the graph representing it.

15. Prove that every simple graph with at least two vertices has two vertices of the same degree.

16. Suppose the average degree of the vertices of a connected graph is exactly 2. How many cycles does G have ? Justify your answer.

17. Show that every connected graph has at least one spanning tree.

18. Show that a graph is a tree if and only if it is minimally connected.

19. If a graph G on n vertices has fewer than 3n – 6 edges, must G be planar ? Justify.

20. Does there exist a digraph G with at least 3 vertices such that no two vertices of G have the same outdegree, but all the vertices have the same indegree ? Justify.

21. Show that a bipartite graph contains no circuit of odd length.

22. Let G be a simple graph, where the minimum degree of a vertex is k. Show that G contains a path of length at least k.

## SECTION – III

Answer **any 6** questions from among the questions **23** to **31**. These questions carry **4** marks **each**.

23. Let T be a tree in which all vertices adjacent to pendant vertices have degree at least 3. Prove that there exists a pair $(v_1, v_2)$ of pendant vertices that have a common adjacent vertex.

24. Show that any connected graph with n vertices and n – 1 edges is a tree.

25. Let G be a simple connected graph with at least 11 vertices. Prove that either G or its complement $\overline{G}$ must be non-planar.

26. Show that in a connected graph G with exactly 2k odd vertices, there exist k edge disjoint subgraphs such that they together contain all edges of G and that each is a unicursal graph.

27. Show that the maximum number of edges in a complete bipartite graph with n vertices is $\lfloor n^2/4 \rfloor$.

28. Let G be a simple graph with n vertices and m edges. Show that if $m > \binom{n-1}{2}$, then G is connected.

29. Let G be a simple graph with 6 vertices. Show that either G or its complement $\overline{G}$ contains $K_3$ as a subgraph.

30. Let G be a simple graph having no isolated vertex and no induced subgraph with exactly two edges. Prove that G is a complete graph.

31. For $n \geq 3$, let G be an n-vertex graph such that every graph obtained by removing one vertex is a tree. Show that G itself is a cycle.

## SECTION – IV

Answer **any 2** questions from among the questions **32** to **35**. These questions carry **15** marks **each**.

32. a) Show that every tree has either one or two adjacent enters.

    b) Show that $K_5$ and $K_{3,3}$ are non-planar.

33. a) Show that a simple graph with n vertices and k components can have at most $\frac{1}{2}(n-k)(n-k+1)$ edges.

    b) Show that a connected graph G is an Euler graph if and only if all vertices of G are of even degree.

34. a) Prove that a digraph G is an Euler digraph iff G is connected and is balanced.

    b) Does there exist a non-planar graph which does not have either of the Kuratowski graphs as a subgraph ? Justify.

35. a) Define the following and give example for each :

        i) Asymmetric digraph

        ii) Complete symmetric digraph

        iii) Complete tournament

    b) Let G be a simple connected planar graph with less than 12 vertices. Prove that G has a vertex of degree at most 4.

    c) Let G be a simple connected planar graph with less than 30 edges. Prove that G has a vertex of degree at most 4.

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